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Resonant scattering can enhance the degree of entanglement

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Abstract

Generation of entanglement between two qubits by scattering an entanglement mediator is discussed. The mediator bounces between the two qubits and exhibits a resonant scattering. It is clarified how the degree of the entanglement is enhanced by the constructive interference of such bouncing processes. Maximally entangled states are available via adjusting the incident momentum of the mediator or the distance between the two qubits, but their fine tunings are not necessarily required to gain highly entangled states and a robust generation of entanglement is possible.

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1. Introduction

Entanglement plays an essential role in the ideas of quantum information, like quantum computation, quantum communication, quantum cryptography and so on [1]. Efficient generation of entanglement hence constitutes an essential element for the realization of such ideas, and various schemes have been proposed. Entanglement would be most simply and naturally generated by a direct interaction between two entities carrying quantum information [2], i.e. between two qubits. There are, however, several setups in which the two qubits are separated from each other, beyond the range of the direct interaction, but an entanglement is to be shared between them. In such a case, a ‘mediator’ would be convenient to entangle the separated qubits, and several schemes have been investigated theoretically [3–11] and experimentally [12, 13].

Among these proposals, we here concentrate on a scheme based on the ‘successive’ interactions of a mediator with two qubits to be entangled [6–13]. Mediator X prepared in a specific state is sent to interact successively one by one with qubits A and B, each of which is

prepared in an appropriate initial state, and then the state of X is measured. If X is found in a specific state after the interactions, we end up with an entanglement between A and B.

It has been somewhat standard in such schemes to assume that mediator X interacts with qubit A(B) for a certain time duration $\tau_{A(B)}$ [6–10, 12], and the generation of a highly entangled state with a high probability is accomplished by tuning these times, given the coupling constants. It is, however, a subtle problem whether the notion of the ‘interaction time’ makes sense or not. It might be valid when the size of a wave packet is small enough compared with the interaction region, but it might not be otherwise. Even in the former case, no rigorous proof has been given explicitly so far, to the best of the present authors’ knowledge.

A rigorous approach to this issue would be to formulate it as a *scattering problem* and such attempts have recently appeared [10, 11]. Qubits A and B are shown to be entangled after *scattering* mediator X. In this approach, the notion of the interaction time is not necessary; actually, such a time is either absent in the stationary-state treatments or automatically given by the physical situation under consideration in the wave-packet scatterings [14]. At this point, it is worth noting that X can be reflected by each of the qubits with a certain probability. This effect is not taken into account in the standard formulations with the interaction times. The wave reflected by B is partially directed to A and some of its portion is reflected back to B. Such a sequence of bounces between A and B gives rise to a resonant scattering of mediator X.

The purpose of the present work is to clarify the effects of the 1D resonant scattering on the entanglement generation³: resonance can enhance the degree of entanglement. It is revealed that the distance d between the two qubits is an important parameter to gain an entanglement efficiently, and maximally entangled states are available with finite probabilities by adjusting d or the incident momentum k of X, in a wide parameter region of the coupling constants. Fine tunings of k or d , however, are not necessarily required and robust generation of highly entangled states is possible.

2. Scattering of a monochromatic wave

We consider the following Hamiltonian in 1D space:

$$H = \frac{p^2}{2m} + g_A(\sigma_+^{(X)}\sigma_-^{(A)} + \sigma_-^{(X)}\sigma_+^{(A)})\delta(x + d/2) + g_B(\sigma_+^{(X)}\sigma_-^{(B)} + \sigma_-^{(X)}\sigma_+^{(B)})\delta(x - d/2). \quad (1)$$

Qubits A and B are placed at $x = -d/2$ and $d/2$, respectively, and the quantum information is encoded in their spin states $|\uparrow\rangle_{A(B)}$ and $|\downarrow\rangle_{A(B)}$, which are flipped by the ladder operators $\sigma_{\mp}^{(A(B))}$. A and B are initially prepared in $|\uparrow\uparrow\rangle_{AB}$ and mediator X polarized in $|\downarrow\rangle_X$ is injected from the left. X, whose position and momentum are represented by the operators x and p , respectively, propagates according to the Hamiltonian (1) and is scattered by A and B (figure 1). Since the interactions between mediator X and qubits A, B preserve the number of spins in the up state $|\uparrow\rangle$ among A, B, and X, if X after the scattering is detected by either of the two detectors on both sides and found in $|\uparrow\rangle_X$, either state of A or B is flipped down and an entangled state is generated (see (13a) and (13b) below). (Possible entanglement generation without resolving the spin state of X by the detector will be discussed in section 6.) Note

³ The resonance effect is not clarified in [10]. In fact, the distance between the two scatterers, d , is not contained in the formulae in [10], while it is the important parameter for the resonance. Reference [11] briefly discusses the entanglement generation in the same system as that discussed in section 5 of this paper, but the parameter kd (which is kx_0 in [11]) is fixed at $kd = \nu\pi$ ($\nu = 1, 2, \dots$), and the effect of the resonant scattering on the entanglement generation is not fully clarified.

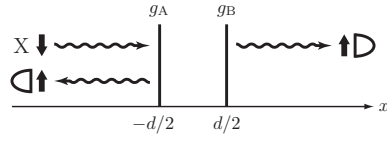


Figure 1. Mediator X prepared in $|\downarrow\rangle_X$ is scattered by qubits A and B placed at $x = -d/2$ and $d/2$, respectively, and initialized in $|\uparrow\uparrow\rangle_{AB}$, and $|\uparrow\rangle_X$ is to be detected by either of the two detectors on both sides, with a certain probability.

that we are looking at an extreme case where the potential barriers are much thinner than the extension of the wave packet of X .

Let us find a scattering state of the Hamiltonian (1),

$$H|E\rangle = E|E\rangle, \quad (2)$$

which would describe the scattering of a well-mono-chromatized incident particle X . We introduce the wavefunctions

$$u_{\downarrow;\uparrow\uparrow}(x) = \langle X(x\downarrow | \otimes_{AB} \langle \uparrow\uparrow | | E \rangle \equiv \langle x\downarrow; \uparrow\uparrow | E \rangle, \text{ etc.} \quad (3)$$

The solution is given in the form

$$u(x) = \begin{cases} I e^{ik(x+d/2)} + R e^{-ik(x+d/2)} & (x < -d/2), \\ T e^{ik(x-d/2)} & (x > d/2) \end{cases} \quad (4)$$

for each spin state, where k is the momentum (apart from \hbar) of X and $E = \hbar^2 k^2 / 2m$.

We are interested in the entanglement generation from the initialized state $|\uparrow\uparrow\rangle_{AB}$ by injecting X in the spin state $|\downarrow\rangle_X$. In such a case, only the three wavefunctions $u_{\downarrow;\uparrow\uparrow}(x)$, $u_{\uparrow;\uparrow\downarrow}(x)$ and $u_{\uparrow;\downarrow\uparrow}(x)$ are involved in the problem. Imposing the continuity condition on the wavefunctions, the scattering problem (2) is solved under the boundary conditions

$$I_{\downarrow;\uparrow\uparrow} = 1, \quad I_{\uparrow;\uparrow\downarrow} = 0, \quad I_{\uparrow;\downarrow\uparrow} = 0 \quad (5)$$

to yield the transmission and reflection amplitudes

$$T_{\downarrow;\uparrow\uparrow} = \frac{t_A t_B e^{ikd}}{1 - r_A r_B e^{2ikd}}, \quad (6a)$$

$$R_{\downarrow;\uparrow\uparrow} = r_A + \frac{t_A^2 r_B e^{2ikd}}{1 - r_A r_B e^{2ikd}}, \quad (6b)$$

$$T_{\uparrow;\uparrow\downarrow} = R_{\uparrow;\uparrow\downarrow} e^{-ikd} = \frac{t_A f_B e^{ikd}}{1 - r_A r_B e^{2ikd}}, \quad (6c)$$

$$T_{\uparrow;\downarrow\uparrow} = R_{\uparrow;\downarrow\uparrow} e^{ikd} = \left(1 + \frac{t_A r_B e^{2ikd}}{1 - r_A r_B e^{2ikd}} \right) f_A e^{ikd}, \quad (6d)$$

where

$$t_{A(B)} = \frac{1}{1 + \Omega_{A(B)}^2}, \quad r_{A(B)} = -\frac{\Omega_{A(B)}^2}{1 + \Omega_{A(B)}^2}, \quad (7a)$$

$$f_{A(B)} = -\frac{i\Omega_{A(B)}}{1 + \Omega_{A(B)}^2}, \quad (7b)$$

and $\Omega_{A(B)} = mg_{A(B)}/\hbar^2 k$. Note that X is not affected by A(B) when they are in the same spin states. The coefficients $t_{A(B)}$ and $r_{A(B)}$ respectively describe the transmission through and the reflection from A(B) without spin flip and $f_{A(B)}$ the transmission/reflection with spin flips, when X is in a different spin state from A(B).

It is interesting to expand the amplitudes in (6a)–(6d) as power series in e^{2ikd} :

$$\begin{aligned} T_{\uparrow;\uparrow\downarrow} &= R_{\uparrow;\uparrow\downarrow} e^{-ikd} \\ &= t_A e^{ikd} (1 + r_B e^{ikd} r_A e^{ikd} + r_B e^{ikd} r_A e^{ikd} r_B e^{ikd} r_A e^{ikd} + \dots) f_B, \end{aligned} \quad (8a)$$

$$\begin{aligned} T_{\uparrow;\downarrow\uparrow} &= R_{\uparrow;\downarrow\uparrow} e^{ikd} \\ &= (1 + t_A e^{ikd} r_B e^{ikd} + t_A e^{ikd} r_B e^{ikd} r_A e^{ikd} r_B e^{ikd} + \dots) f_A e^{ikd}, \end{aligned} \quad (8b)$$

etc., where e^{ikd} represents the phase factor gained for the rightwards/leftwards propagation of X between A and B, and each term of the expansions reveals how X goes back and forth between A and B, before flipping its spin by the interaction with A or B.

It is also worth looking at the coefficients $t_{A(B)}$, $r_{A(B)}$ and $f_{A(B)}$ for the single potential A(B) and comparing the present approach with the ordinary formulation with the interaction times [6–10, 12]. When we concentrate on a transmitted particle (like in the approach with the interaction time), a state of X and A, e.g. $|\downarrow; \uparrow\rangle_{XA}$, is ‘rotated’ after the transmission of X through potential A like

$$|\downarrow; \uparrow\rangle_{XA} \rightarrow t_A |\downarrow; \uparrow\rangle_{XA} + f_A |\uparrow; \downarrow\rangle_{XA}, \quad (9)$$

apart from the normalization. The counterpart in the approach with the interaction time reads [8, 9]

$$|\downarrow; \uparrow\rangle_{XA} \rightarrow \cos g_A \tau_A |\downarrow; \uparrow\rangle_{XA} - i \sin g_A \tau_A |\uparrow; \downarrow\rangle_{XA}, \quad (10)$$

and there exists the connection between the two formulations:

$$\cos g_A \tau_A \leftrightarrow \frac{1}{\sqrt{1 + \Omega_A^2}}, \quad \sin g_A \tau_A \leftrightarrow \frac{\Omega_A}{\sqrt{1 + \Omega_A^2}}. \quad (11)$$

These relations show that $g_A \tau_A$ ranges from 0 to $\pi/2$ and a higher/lower momentum corresponds to a shorter/longer interaction time.

In practise, however, one should also take account of the probability for such event, transmission, to occur. To achieve the complete flip $\sin g_A \tau_A = 1$, for instance, the correspondence (11) suggests that $\Omega_A \rightarrow \infty$ is required. But the probability of the transmission

$$|t_A|^2 + |f_A|^2 = \frac{1}{1 + \Omega_A^2} \quad (12)$$

vanishes in this limit, meaning that the complete flip is not possible. A larger rotation angle $g_A \tau_A$ requires a larger Ω_A but it is available with a smaller probability. This restricts the applicability of the formulation with the interaction times, at least for the delta-shaped potential.

3. Generation of entanglement

Now we are ready to discuss the entanglement generation. If a transmitted/reflected particle in $|\uparrow\rangle_X$ is detected by the detector on the right/left, an entangled state

$$|\Psi_i\rangle_{AB} \propto T_{\uparrow;\uparrow\downarrow} |\uparrow\downarrow\rangle_{AB} + T_{\uparrow;\downarrow\uparrow} |\downarrow\uparrow\rangle_{AB} \quad (13a)$$

(for the former case) or

$$|\Psi_T\rangle_{AB} \propto R_{\uparrow;\uparrow\downarrow}|\uparrow\downarrow\rangle_{AB} + R_{\uparrow;\downarrow\uparrow}|\downarrow\uparrow\rangle_{AB} \quad (13b)$$

(for the latter) is established. In the present model, the concurrences [15] of these states are the same,

$$C = \frac{2|T_{\uparrow;\uparrow\downarrow}T_{\uparrow;\downarrow\uparrow}|}{|T_{\uparrow;\uparrow\downarrow}|^2 + |T_{\uparrow;\downarrow\uparrow}|^2} = \frac{2|R_{\uparrow;\uparrow\downarrow}R_{\uparrow;\downarrow\uparrow}|}{|R_{\uparrow;\uparrow\downarrow}|^2 + |R_{\uparrow;\downarrow\uparrow}|^2} = \frac{2a}{1+a^2}, \quad (14a)$$

$$a = \left| \frac{T_{\uparrow;\downarrow\uparrow}}{T_{\uparrow;\uparrow\downarrow}} \right| = \left| \frac{R_{\uparrow;\downarrow\uparrow}}{R_{\uparrow;\uparrow\downarrow}} \right| = \frac{\Omega_A}{\Omega_B} \sqrt{1 + 4\Omega_B^2(1 + \Omega_B^2) \sin^2 kd}, \quad (14b)$$

and so are the probabilities of the generations of the entangled states (13a) and (13b), namely the probability of detecting a transmitted particle in the $|\uparrow\rangle_X$ state and that of detecting a reflected one,

$$\begin{aligned} P &= |T_{\uparrow;\uparrow\downarrow}|^2 + |T_{\uparrow;\downarrow\uparrow}|^2 = |R_{\uparrow;\uparrow\downarrow}|^2 + |R_{\uparrow;\downarrow\uparrow}|^2 \\ &= \frac{\Omega_A^2 + \Omega_B^2 + 4\Omega_A^2\Omega_B^2(1 + \Omega_B^2) \sin^2 kd}{(1 + \Omega_A^2 + \Omega_B^2)^2 + 4\Omega_A^2\Omega_B^2(1 + \Omega_A^2)(1 + \Omega_B^2) \sin^2 kd}. \end{aligned} \quad (15)$$

Note that the probability of *either* of the two detectors detecting X in the state $|\uparrow\rangle_X$ is given by $2P$, and therefore the maximal value of P is $1/2$.

Note here that the distance between the two qubits, d , enters the formulae through the exponential factor e^{2ikd} , which is responsible for the resonant scattering. See figure 2, where the resonance structures are observed in the momentum dependences of the concurrence C and the probability P . An apparent feature is that the oscillations of C and P are out of phase: one can see a trade-off between C and P . But still, a highly entangled state is generated with a finite (non-vanishing) probability. At the same time, fine tunings of parameters are not necessarily required. For example, let us look at a case where $g_A = g_B$. Figure 2(c) shows that C and P do not oscillate strongly for large k and a high concurrence is available in a wide range of the incident momentum k . This is because the oscillating factor e^{2ikd} represents the bouncing of X between A and B and it always accompanies the reflection coefficients r_A and r_B , which are reduced for large momentum k and suppress the oscillations.

Let us look more closely at the formulae and see how a highly entangled state is acquired by the resonant tunnelling. Figure 2(d) shows that the series expansions in (8a) and (8b) converge very quickly and the first two contributions, i.e. the no- and one-bounce processes, are important. For $g_A = g_B$, the no-bounce process, i.e. the first terms in (8a) and (8b), yields $|T_{\uparrow;\uparrow\downarrow}| < |T_{\uparrow;\downarrow\uparrow}|$ ($|R_{\uparrow;\uparrow\downarrow}| < |R_{\uparrow;\downarrow\uparrow}|$). If the momentum k is adjusted so as to satisfy $kd = \nu\pi$ ($\nu = 1, 2, \dots$), the one-bounce process, i.e. the second terms in (8a) and (8b), interferes with the no-bounce process *constructively* for the $|\uparrow;\uparrow\downarrow\rangle$ component but *destructively* for the $|\uparrow;\downarrow\uparrow\rangle$ component (since $t_{A(B)} > 0$ while $r_{A(B)} < 0$). As a result, the difference between $|T_{\uparrow;\uparrow\downarrow}|$ and $|T_{\uparrow;\downarrow\uparrow}|$ ($|R_{\uparrow;\uparrow\downarrow}|$ and $|R_{\uparrow;\downarrow\uparrow}|$) is reduced by the interference and the concurrence is made close to unity. In fact, the ratio a introduced in (14b) is evaluated up to the one-bounce contribution to be $a = 1 + \delta a = 1 + \Omega^6/(1 + 2\Omega^2 + 2\Omega^4)$ under the conditions $g_A = g_B$ and $kd = \nu\pi$ ($\nu = 1, 2, \dots$), and the deviation of the concurrence from unity is about $(\delta a)^2/2$, which is very small for $\Omega (= \Omega_A = \Omega_B) \lesssim 1$ (almost perfect compensation). Other processes with more bounces complete the exact unit concurrence. Note that, in figure 2(c), where $g_A = g_B = 3[\hbar^2\pi/md]$, the condition $\Omega \lesssim 1$ corresponds to $k \gtrsim 3[\pi/d]$.

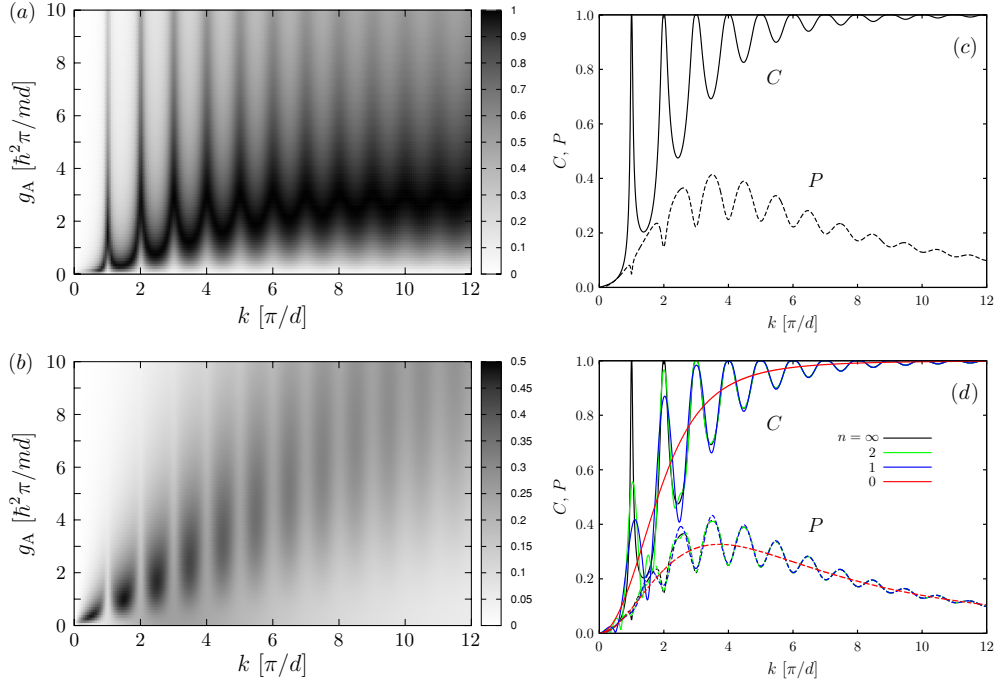


Figure 2. (a) The concurrence C given in (14a)–(14b) and (b) the probability P given in (15) as functions of k and g_A when $g_B = 3[\hbar^2\pi/md]$. (c) The concurrence C (solid) and the probability P (dashed) as functions of the incident momentum k when $g_A = g_B = 3[\hbar^2\pi/md]$. (d) The same as (c) but only the contributions up to the $(n + 1)$ th term in each of the series expansions (8a) and (8b) are taken into account, where n is the number of bounces between A and B.

(This figure is in colour only in the electronic version)

4. Optimization

Let us next survey the whole parameter space and discuss how to optimize the concurrence C in (14a)–(14b) and the probability P in (15). There are essentially three independent (dimensionless) parameters $\Omega_{A(B)} = mg_{A(B)}/\hbar^2k$ and kd .

We first look at the probability (15). It is possible to show that it is optimal for a given pair (Ω_A, Ω_B) when the resonance condition

$$\sin^2 kd = 1 \quad (16)$$

is satisfied. The probability P under this condition is plotted in figure 3(a) as a function of Ω_A and Ω_B , together with the concurrence C . The maximum value of the probability P is given by

$$P_{\max} = \frac{1}{2} \quad \text{at} \quad (\Omega_A, \Omega_B) = (1/\sqrt{2}, \infty), \quad (17)$$

while the unit concurrence $C = 1$ under the resonance condition (16) is achieved when

$$\Omega_A = \frac{\Omega_B}{1 + 2\Omega_B^2}. \quad (18)$$

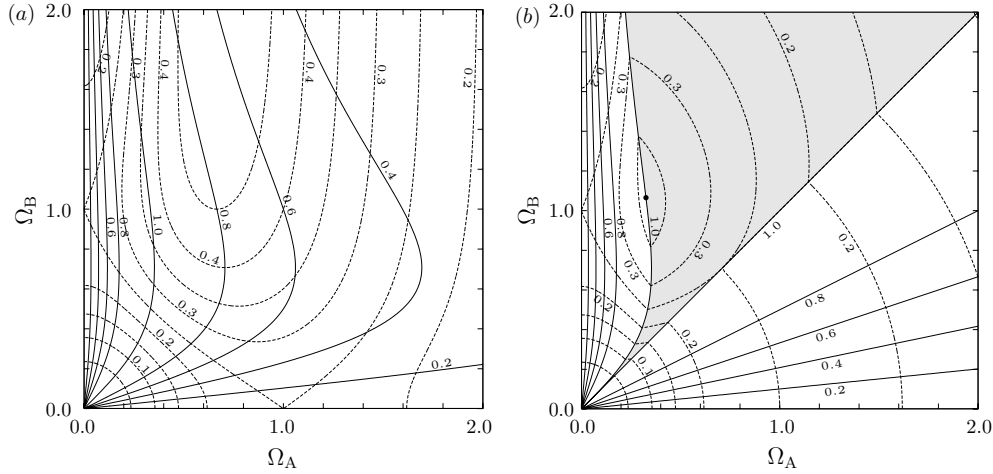


Figure 3. (a) Contour plots of the probability P (dashed) given in (15) and the concurrence C (solid) given in (14a)–(14b) as functions of Ω_A and Ω_B under the resonance condition $\sin^2 kd = 1$. The probability P takes its maximum value $P_{\max} = 1/2$ at $(\Omega_A, \Omega_B) = (1/\sqrt{2}, \infty)$, and the concurrence is unity $C = 1$ on the line $\Omega_A = \Omega_B/(1 + 2\Omega_B^2)$. (b) Contour plots of the optimal concurrence C for given (Ω_A, Ω_B) (solid) and the corresponding probability P (dashed) as functions of Ω_A and Ω_B . The unit concurrence $C = 1$ is available in the grey region $\Omega_B/(1 + 2\Omega_B^2) \leq \Omega_A \leq \Omega_B$. In the left region $\Omega_A \leq \Omega_B/(1 + 2\Omega_B^2)$, the concurrence C is optimal when $\sin^2 kd = 1$, and in the right region $\Omega_A \geq \Omega_B$, it is optimal when $\sin^2 kd = 0$. The optimal probability P for the unit concurrence $C = 1$ is realized at the point indicated by a dot, $(\Omega_A, \Omega_B) \simeq (0.33, 1.07)$, and is given by $P_{\text{opt}} \simeq 0.37$.

If one does not stick to a high probability, the concurrence C is optimized as follows. The concurrence (14a)–(14b) takes its maximum $C = 1$ at $a = 1$ and decreases monotonically as a leaves this optimal point. Since

$$\frac{\Omega_A}{\Omega_B} \leq a \leq \frac{\Omega_A}{\Omega_B} (1 + 2\Omega_B^2) \quad (19)$$

for a given pair (Ω_A, Ω_B) , the unit concurrence $C = 1$ (i.e. $a = 1$) requires

$$\frac{\Omega_A}{\Omega_B} \leq 1 \leq \frac{\Omega_A}{\Omega_B} (1 + 2\Omega_B^2), \quad \sin^2 kd = \frac{\Omega_B^2 - \Omega_A^2}{4\Omega_A^2 \Omega_B^2 (1 + \Omega_B^2)}. \quad (20)$$

See figure 3(b), where the region for the unit concurrence $C = 1$ is shown, together with the corresponding probability P . In particular, the unit concurrence $C = 1$ is always available when $g_A = g_B$, by adjusting the incident momentum k or the distance between the two qubits, d . The optimal probability P for the unit concurrence $C = 1$ is realized at the point indicated by a dot in figure 3(b), which is evaluated to be

$$P_{\text{opt}} \simeq 0.37 \quad (21a)$$

at

$$\Omega_B = \sqrt{\frac{1 + \sqrt[3]{37 - 3\sqrt{114}} + \sqrt[3]{37 + 3\sqrt{114}}}{6}} \simeq 1.07, \quad (21b)$$

$$\Omega_A = \frac{\Omega_B}{1 + 2\Omega_B^2} \simeq 0.33, \quad \sin^2 kd = 1. \quad (21c)$$

5. Another model

Similar analyses are possible for the Hamiltonian

$$H = \frac{p^2}{2m} + g_A \boldsymbol{\sigma}^{(X)} \cdot \boldsymbol{\sigma}^{(A)} \delta(x + d/2) + g_B \boldsymbol{\sigma}^{(X)} \cdot \boldsymbol{\sigma}^{(B)} \delta(x - d/2). \quad (22)$$

This type of interaction is considered in [10, 11] and would have physical applications. In this case, the amplitudes are given by

$$T_{\downarrow;\uparrow\uparrow} = \frac{\tilde{t}_A \tilde{t}_B e^{ikd}}{1 - \tilde{r}_A \tilde{r}_B e^{2ikd}}, \quad (23a)$$

$$R_{\downarrow;\uparrow\uparrow} = \tilde{r}_A + \frac{\tilde{t}_A^2 \tilde{r}_B e^{2ikd}}{1 - \tilde{r}_A \tilde{r}_B e^{2ikd}}, \quad (23b)$$

$$T_{\uparrow;\uparrow\downarrow} = \frac{\tilde{t}_A e^{ikd}}{1 - \tilde{r}_A \tilde{r}_B e^{2ikd}} f_B \left(1 + \frac{r'_A t_B e^{2ikd}}{1 - r'_A r_B e^{2ikd}} \right), \quad (23c)$$

$$R_{\uparrow;\uparrow\downarrow} = \frac{\tilde{t}_A e^{ikd}}{1 - \tilde{r}_A \tilde{r}_B e^{2ikd}} f_B \frac{t'_A e^{ikd}}{1 - r'_A r_B e^{2ikd}}, \quad (23d)$$

$$T_{\uparrow;\downarrow\uparrow} = \left(1 + \frac{\tilde{t}_A \tilde{r}_B e^{2ikd}}{1 - \tilde{r}_A \tilde{r}_B e^{2ikd}} \right) f_A \frac{t'_B e^{ikd}}{1 - r_A r'_B e^{2ikd}}, \quad (23e)$$

$$R_{\uparrow;\downarrow\uparrow} = \left(1 + \frac{\tilde{t}_A \tilde{r}_B e^{2ikd}}{1 - \tilde{r}_A \tilde{r}_B e^{2ikd}} \right) f_A \left(1 + \frac{t_A r'_B e^{2ikd}}{1 - r_A r'_B e^{2ikd}} \right), \quad (23f)$$

with

$$t_{A(B)} = \frac{1 - i\Omega_{A(B)}}{(1 + i\Omega_{A(B)})(1 - 3i\Omega_{A(B)})}, \quad (24a)$$

$$r_{A(B)} = \frac{i\Omega_{A(B)}(1 + 3i\Omega_{A(B)})}{(1 + i\Omega_{A(B)})(1 - 3i\Omega_{A(B)})}, \quad (24b)$$

$$f_{A(B)} = -\frac{2i\Omega_{A(B)}}{(1 + i\Omega_{A(B)})(1 - 3i\Omega_{A(B)})}, \quad (24c)$$

$$t'_{A(B)} = \frac{1}{1 + i\Omega_{A(B)}}, \quad r'_{A(B)} = -\frac{i\Omega_{A(B)}}{1 + i\Omega_{A(B)}}, \quad (24d)$$

and

$$\tilde{t}_{A(B)} = t_{A(B)} + \Sigma_{A(B)}, \quad \tilde{r}_{A(B)} = r_{A(B)} + \Sigma_{A(B)}, \quad (25a)$$

$$\Sigma_{A(B)} = \frac{f_{A(B)}^2 r'_{B(A)} e^{2ikd}}{1 - r_{A(B)} r'_{B(A)} e^{2ikd}}. \quad (25b)$$

In contrast to the previous model, X is affected by potential A(B) even when X is in the same spin state as A(B). $t'_{A(B)}$ and $r'_{A(B)}$ are the coefficients for the transmission through and the reflection from the single potential A(B) without spin flip when X is in the same spin state as A(B), $t_{A(B)}$ and $r_{A(B)}$ are those when X is in a different spin state from A(B), and $f_{A(B)}$ describes the transmission and the reflection with spin flip when X is in a different spin

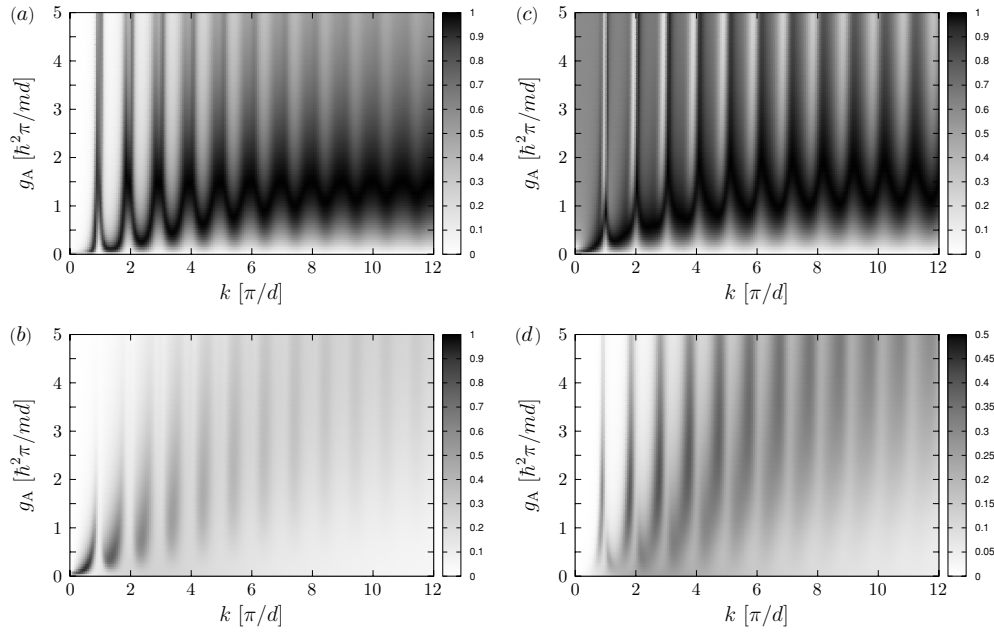


Figure 4. The concurrences (above: (a), (c)) and the probabilities (below: (b), (d)) for the Hamiltonian (22) as functions of k and g_A with $g_B = 1.5[\hbar^2\pi/md]$, when a reflected particle is detected (left: (a), (b)) and when a transmitted particle is detected (right: (c), (d)).

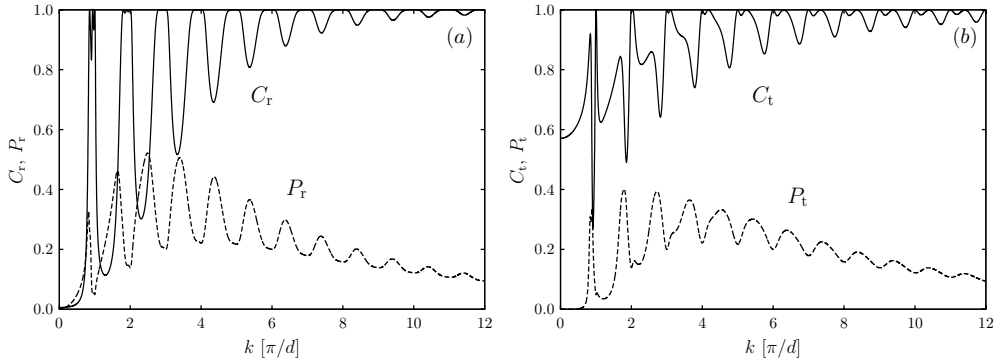


Figure 5. The concurrences (solid) and the probabilities (dashed) for the Hamiltonian (22) as functions of the incident momentum k with $g_A = g_B = 1.5[\hbar^2\pi/md]$, when a reflected particle is detected (a) and when a transmitted particle is detected (b).

state from A(B). For this model, the concurrence C_t and the probability P_t by detecting a transmitted particle in $|\uparrow\rangle_X$ on the right are different from C_r and P_r by detecting a reflected one on the left. Again, the denominators in (23a)–(23f) and (25b) represent bouncing of X between A and B (which becomes clear by expanding them as power series in e^{2ikd}) and give rise to the resonant tunnelling. Similar resonance structures to the previous example are observed in the concurrences and the probabilities, as shown in figures 4 and 5. The oscillations of the concurrences and the probabilities are reduced for a large momentum k and a robust entanglement generation is available (figure 5), due to the same mechanism as in the previous example.

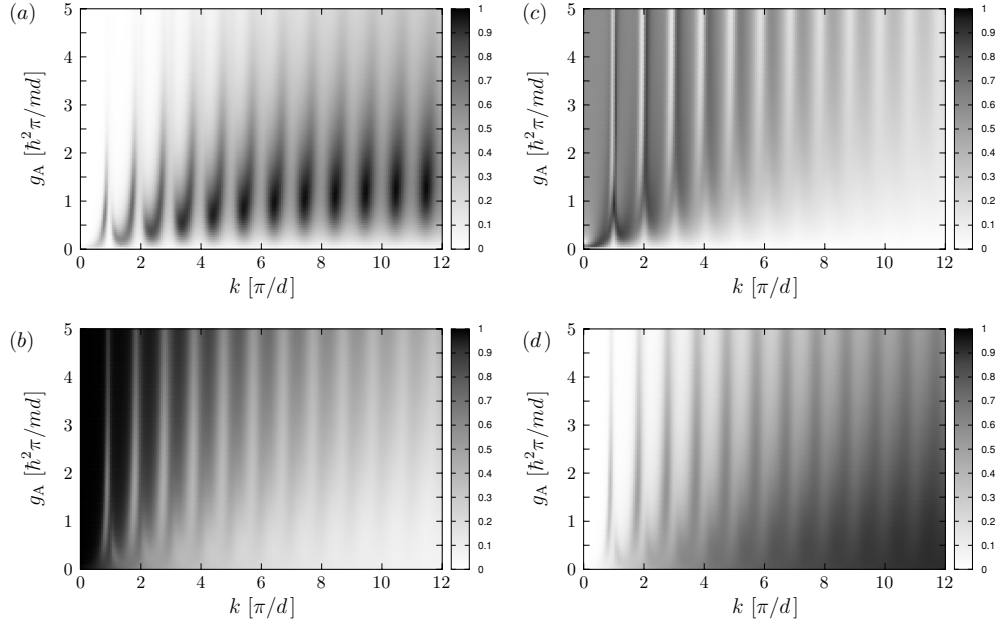


Figure 6. The concurrences (above: (a), (c)) and the probabilities (below: (b), (d)) for the Hamiltonian (22) as functions of k and g_A with $g_B = 1.5\hbar^2\pi/md$, when a reflected particle (left: (a), (b)) or a transmitted one (right: (c), (d)) is detected regardless of its spin state.

6. Entanglement generation without resolving the spin state of X

In the above schemes, the entangled state (13a) or (13b) is generated by detecting a transmitted or reflected particle *in the spin state* $|\uparrow\rangle_X$. Such a detector that is able to resolve the spin state of the scattered particle is required and might be difficult to implement in practise. Fortunately, however, a (not maximally but) highly entangled state is still available by just detecting a transmitted or reflected particle *without resolving its spin state*.

A reflected particle is detected on the left (regardless of its spin state) with probability

$$\tilde{P}_r = |R_{\downarrow;\uparrow\uparrow}|^2 + |R_{\uparrow;\uparrow\downarrow}|^2 + |R_{\uparrow;\downarrow\uparrow}|^2 \quad (26)$$

and we end up with a state of A and B,

$$\rho_r \propto |R_{\downarrow;\uparrow\uparrow}|^2 |\uparrow\uparrow\rangle\langle\uparrow\uparrow|_{AB} + (R_{\uparrow;\uparrow\downarrow}|\uparrow\downarrow\rangle_{AB} + R_{\uparrow;\downarrow\uparrow}|\downarrow\uparrow\rangle_{AB})(R_{\uparrow;\uparrow\downarrow}^* \langle\uparrow\downarrow| + R_{\uparrow;\downarrow\uparrow}^* \langle\downarrow\uparrow|). \quad (27)$$

This is a mixed state unlike (13b), but still a certain amount of entanglement is created between A and B, which is evaluated by the concurrence [15]

$$\tilde{C}_r = \frac{2|R_{\uparrow;\uparrow\downarrow}R_{\uparrow;\downarrow\uparrow}|}{|R_{\downarrow;\uparrow\uparrow}|^2 + |R_{\uparrow;\uparrow\downarrow}|^2 + |R_{\uparrow;\downarrow\uparrow}|^2}. \quad (28)$$

Similar formulae for the state ρ_t , the concurrence \tilde{C}_t , and the probability \tilde{P}_t are available for the detection of a transmitted particle on the right. Note the conservation of probability $\tilde{P}_t + \tilde{P}_r = 1$.

It is apparent that the concurrences \tilde{C}_t and \tilde{C}_r are less than those by the spin-resolved detections, (14a)–(14b), while the probabilities \tilde{P}_t and \tilde{P}_r are greater than the counterparts in (15). Let us look at them in detail for the model analysed in section 5. Figures 6 and 7 show

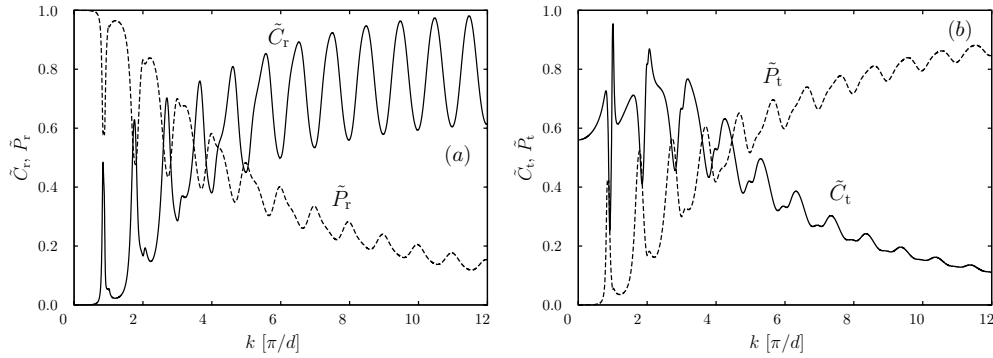


Figure 7. The concurrences (solid) and the probabilities (dashed) for the Hamiltonian (22) as functions of the incident momentum k with $g_A = g_B = 1.5[\hbar^2\pi/md]$, when a reflected particle (a) or a transmitted one (b) is detected regardless of its spin state.

that a highly entangled state (although not maximally entangled) is generated with a finite probability just by detecting a scattered particle (especially, a reflected one) without resolving its spin state.

Note that the contributions of the processes with one reflection to the reflection amplitudes (23b), (23d) and (23f) read $R_{\downarrow;\uparrow\uparrow} \sim i\Omega_A$, $R_{\uparrow;\uparrow\downarrow} \sim -2i\Omega_B e^{2ikd}$, and $R_{\uparrow;\downarrow\uparrow} \sim -2i\Omega_A$ for large incident momentum k , and hence, $\tilde{C}_r \sim 8\Omega_A\Omega_B / (5\Omega_A^2 + 4\Omega_B^2)$ ($=8/9$ if $\Omega_A = \Omega_B$). Large concurrence is available for large momentum k and is enhanced more by the resonance, as figure 7(a) shows. This result would promise a physical implementation of the scheme with a less restrictive requirement on the detector.

7. Summary

In this paper we have investigated the entanglement generation by the resonant scattering. The resonance effects are clarified and the optimization of the entanglement generation is discussed. The interference of the bouncing and non-bouncing processes can enhance entanglement. Maximally entangled states are available with finite probabilities in a wide parameter region. The degree of entanglement is optimized by adjusting the momentum of the mediator, but its fine tuning is not necessarily required. Furthermore, it has been clarified that the detectors without spin resolution can be used for the entanglement generation.

One of the interesting future subjects would be to clarify the effect of the size of a wave packet on the generation of entanglement. Scattering of plane waves is discussed in this paper, but the models are solvable also for wave packets [14]. It is possible to discuss the resonant scattering fully dynamically. Introduction of the width of the potential would also be interesting to explore the validity of the ordinary formulation with the interaction times.

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